Chapter 8
Supporting Mathematics Teacher Educators' Growth and Development Through Communities of Practice

Dana Olanoff, Joanna O. Masingila, and Patrick M. Kimani

8.1 Background

Mathematics teacher educators (MTEs) play a significant role in helping prospective teachers (PTs) to develop the mathematical knowledge that they need for teaching. However, research has shown that the majority of MTEs in the United States have little experience teaching students at the level of mathematics that they are preparing PTs to teach (e.g. elementary school) and that they receive little to no training or support either in their preparation programmes or in their jobs (Masingila, Olanoff, & Kwaka, 2012). In order to attempt to improve our teaching of mathematics content courses for prospective elementary school teachers, two novice MTEs (Dana and Patrick) worked with an experienced MTE (Joanna) as part of a mentored teaching experience in the Future Professoriate Program at Syracuse University (SU), where Dana and Patrick were graduate students and Joanna was a faculty member. Through this programme, graduate students can earn a Certificate in University Teaching by engaging in professional development experiences, including attending and presenting at conferences, preparing a portfolio, and participating in a mentored teaching experience. We designed Dana and Patrick's mentored teaching experience around our teaching of two mathematics content courses for PTs. We chose to study our teaching critically and form a community of practice (CoP) to support one another in improving our teaching and developing mathematical knowledge for teaching teachers (MKTT).

D. Olanoff (✉)
Widener University, Chester, PA, USA
e-mail: dolanoff@widener.edu

J. O. Masingila
Syracuse University, Syracuse, NY, USA

P. M. Kimani
Glendale Community College, Glendale, AZ, USA

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We each entered into the teaching experience with different backgrounds and somewhat different motivations. Dana had been prepared as a secondary school mathematics teacher and had taught middle/high school mathematics for 1 year before enrolling in a PhD programme in mathematics education. At the point of the study, she had several years' experience teaching mathematics at the undergraduate level, including 1 year teaching mathematics content courses for PTs. However, she was not particularly happy with how she had taught the courses previously and wanted to do better at creating a supportive environment for PTs' learning. She had been studying how teachers were encouraged to reflect on their practice, and she wanted to incorporate this reflection into her own teaching.

Patrick had been prepared as a secondary school mathematics teacher in Kenya and had several years of experience teaching mathematics at the undergraduate level at SU and 4 years of experience teaching at a community college. His only experience with mathematics content courses for PTs came from an internship where he observed and assisted another instructor in teaching one of the courses. From the internship, he realised that if he wanted to be successful in creating opportunities for PTs to think deeply about elementary school mathematics, he needed to think carefully about creating and facilitating a learning environment that was different from what he had done in his teaching career to that point. In order to achieve this deep thinking, it would be helpful to engage with others who were also trying to do the same thing.

Joanna was an experienced MTE who had taught mathematics content courses for many years and had also taught mathematics in grades 7–12 for 6 years before pursuing a doctoral degree in mathematics education. She was involved in designing the two mathematics content courses at SU and was the course coordinator and a co-author of the textbook used to teach the two courses. Her motivation for participating in the research work was to mentor novice MTEs in learning to support PTs in developing mathematics knowledge for teaching (MKT), to think more deeply and gain understanding about what knowledge is needed to support PTs in learning via problem solving, and to add to her own MKTT.

8.2 Forming the Community of Practice

In order to facilitate our mentored teaching experience, we decided to form a CoP. Wenger, McDermott, and Snyder (2002) defined CoPs as "groups of people who share a concern, a set of problems, or a passion about a topic, and who deepen their knowledge and expertise in this area by interacting on an ongoing basis" (p. 7). Through research and participating in a CoP, members develop and articulate new knowledge in response to questions and problems they have about their practice.

A CoP offers a platform for its members to engage in negotiating shared understandings, learning, meaning-making, and identity. Wenger (1998) identified three dimensions of the community which has practice as the source of coherence: (a) CoP members interact with one another and determine norms and relationships

through *mutual engagement*, (b) CoP members are held together by their understanding of a sense of *joint enterprise*, and (c) CoP members seek to produce, over time, a *shared repertoire* of communal resources (e.g. language, routines, artefacts, and stories). Hezemans and Ritzen (2005) studied a CoP in which they participated at their university, and they argued that CoPs can be "a place where the innovative energy of an organization is bundled: communities then perform an important role of adding value to the process of making the strategic policy operational and creating new and innovative solutions" (p. 46). We believed that forming a CoP would allow us to work together to gain knowledge about and improve our teaching of mathematics content courses for PTs.

For us, the fact that Dana and Patrick needed to have a mentored teaching experience with Joanna as the mentor provided the basis for our mutual engagement. We were each invested in the project, and our joint enterprise of improving our MKTT, as we all were interested in improving our teaching of mathematics content courses for prospective teachers. Through our work, we developed a shared repertoire of lesson plans, reflective memos, meetings, and ways of interacting with one another.

8.3 Theoretical Framings

Our CoP revolved around reflecting on the process of teaching mathematics content courses for PTs, part of our shared repertoire. We guided our reflections around research on reflection and inquiry, as well as research on MKT. Below, we briefly review the literature that guided our work.

8.3.1 Reflection and Inquiry

Chapman (2008) states that reflection on their own teaching is an inherent part of the work of teacher educators, with this reflection involving "examining, framing, and attempting to solve the dilemmas of classroom practice; and being aware of and questioning the assumptions and values [they bring] to teaching" (p. 121). We chose to situate our reflection using a position of *inquiry as stance* (Cochran-Smith & Lytle, 1999). Through this process, groups of teachers engage in joint construction of knowledge through conversation and other forms of collaborative analysis and interpretation. Through talk and writing, they make their tacit knowledge more visible, call into question assumptions about common practices, and generate data that make possible the consideration of alternatives (p. 294).

Researchers such as Cochran-Smith (2003) and Webb, Pachler, Mitchell, and Herrington (2007) propose that teacher educators take inquiry as a stance in examining "the enterprise of teaching, schooling, and teacher education" (Cochran-Smith, 2003, p. 5). Teacher educators in an inquiry community "generate local knowledge, envision and theorize their practice, and interpret and interrogate the

theory and research of others" (Cochran-Smith & Lytle, 1999, p. 289); they produce the knowledge they need "to teach well … when they treat their work as a site for intentional investigation at the same time they treat the knowledge and theory produced by others as generative material for interrogation and interpretation" (Cochran-Smith, 2003, p. 16). Inquiry as stance is critical in examining both one's own work and the work of others. We used an inquiry as stance framing for examining our teaching practice and the practice of the members of our CoP; inquiry as stance provided a framing for our joint enterprise (Wenger, 1998) of developing MKTT.

In studying our practice through the CoP, we took on the role of "reflective practitioners" (Schön, 1983). Schön defines two ways to reflect on one's practice: *reflection on action* and *reflection in action*. The former refers to ways in which members of a community of practice (in our case, mathematics teacher educators) reflect on past experiences with the intention of refining their work to achieve their instructional goals, while the latter refers to "thinking on your feet" (Schön, 1983, p. 54), the reflection that occurs while one is in the process of teaching. Because part of our CoP focused on peer observation, we were able to add another type of reflection to our process: *reflection on the actions of others*. We observed each other's teaching both directly and through reading their memos, and we were able to think deeply about choices that they and we made and reflect on these decisions. We were bound together in mutual engagement (Wenger, 1998) through our reflective practitioner roles, and our reflections became part of our shared repertoire.

Students learn through engagement in tasks (Hiebert & Wearne, 1993); likewise, teacher educators learn through engaging with the tasks of teaching (Zaslavsky, 2005, 2007). Zaslavsky (2008) illustrates how both learners and facilitators learn through working with tasks. She proposes that teacher educators learn or construct knowledge by repeatedly participating in a cycle of designing or modifying tasks, supporting learners while they engage in tasks, and reflecting on learners' work. We modelled our CoP around this process, as we continually discussed and sometimes modified the tasks we used in our courses, facilitated the enactment of the tasks, and then reflected on both our enactments of the tasks and the enactments of our group members through memos, creating a shared repertoire (Wenger, 1998) of communal resources.

8.3.2 Mathematical Knowledge for Teaching

Research on teachers' knowledge has flourished following Shulman's (1986) presidential address at the 1985 American Educational Research Association's annual meeting, where he introduced the idea of pedagogical content knowledge (PCK). This knowledge, which Shulman called the "missing paradigm" (p. 6) in research on teaching, linked knowledge of teaching pedagogy with knowledge of the specific content that was taught. As a result of Shulman's speech and the research that followed, Ball and her colleagues introduced the term *mathematical knowledge for*

teaching (MKT) (e.g. Ball & Bass, 2002; Hill & Ball, 2004), which describes the *mathematical* knowledge required by the work of teaching.

Ball and her colleagues (Ball, Thames, & Phelps, 2008) developed a framework for MKT in which they sought to expand Shulman's descriptions of content knowledge and pedagogical content knowledge to include subcategories of the mathematical knowledge that teachers need to know. They broke mathematical content knowledge into three subcategories: *common content knowledge (CCK)*, *specialised content knowledge (SCK)*, and *horizon content knowledge*. The first of these categories, CCK, refers to the mathematical knowledge that everyone needs to know, whereas SCK refers to the mathematical knowledge that is unique to the work of teachers. Examples of SCK include being able to evaluate student algorithms to determine their validity, explaining why we invert and multiply when we divide fractions, and understanding and being able to correctly use mathematical vocabulary. While these types of knowledge may be found in people other than teachers, the researchers argue that this knowledge is necessary for teachers, but is generally not needed by the typical learner of mathematics. Knowledge at the mathematical horizon involves knowing how mathematical topics are related throughout the curriculum and across year levels beyond that at which one is currently teaching.

The researchers (Ball et al., 2008) similarly broke pedagogical content knowledge into *knowledge of content and students (KCS)*, *knowledge of content and teaching (KCT)*, and *knowledge of the curriculum*. KCS combines knowing about students with knowing about mathematics and includes understanding students' reasoning and knowing common errors and misconceptions that students will have with specific material. KCT involves an understanding of sequencing of topics and the power and value of different mathematical representations. Knowledge of the curriculum involves knowing how a curriculum fits together and is related to knowledge at the mathematical horizon.

While much research has been done about the mathematical knowledge needed for teaching, significantly less research has looked at the mathematical knowledge needed by MTEs to help PTs develop MKT (Castro-Superfine & Li, 2014). A number of researchers have determined that there is a category of knowledge needed by teacher educators that goes beyond the knowledge needed by their students (PTs) (e.g. Jaworski, 2008; Perks & Prestage, 2008; Rider & Lynch-Davis, 2006; Zaslavsky & Leikin, 2004). Additionally, many frameworks for teacher educator knowledge provide a structure similar to a framework for teacher knowledge (Olanoff, Welder, Prasad, & Castro Superfine, 2018). In looking at the parallels between the ways in which MTEs learn and develop knowledge and the ways in which mathematics teachers learn and develop knowledge, we hypothesised that mathematical knowledge for teaching teachers (MKTT) would have similar knowledge categories to the MKT framework discussed above (Ball et al., 2008).

The initial joint enterprise of our CoP involved developing our MKTT. However, as we progressed through the semester, we added the additional enterprise of studying portions of our shared repertoire in order to better understand some of the aspects of MKTT we developed and how our CoP helped us in this enterprise. Thus, our CoP became dual purpose: to develop our MKTT (see, e.g., Hiebert, 2013; Van

Zoest, Moore, & Stockero, 2006 for other examples of this type of CoP) and also to research what we had learned (see Arslan, Van Zoest, & Ruk, 2017 for another example of a research CoP). In the remainder of this chapter, we will detail how we studied our collected data and what we learned about our knowledge development and the role that the CoP played in this development.

8.4 Our CoP Processes

We formed our CoP during the 2007–2008 academic year, when we each taught a section of the same two mathematics content courses for PTs during two consecutive semesters. During the first semester, the course content focused on whole number and operations, number theory, probability and statistics, and functions. The content for the second semester course focused on rational numbers, geometry, and measurement. These courses were taught using the same textbook and general lesson plans and with an emphasis on PTs learning mathematics via cooperative problem-solving. PTs worked together in small groups to solve problems with the goal of developing deeper understandings of the K-8 level (elementary and middle years) mathematics and their own MKT. The job of the instructors of the courses was to help facilitate the PTs' problem-solving and knowledge development. During the course of the two semesters, Dana and Patrick, the two novice MTEs, observed Joanna, the experienced MTE, teach her class before teaching their own classes later in the day. Joanna also observed Dana and Patrick's classes several times during each semester. In order for us to reflect on our teaching and observations, and to have a record of these reflections, each of us wrote a memo after each lesson session. We met briefly before each lesson and weekly after we had taught all of the lessons for the week to discuss our observations, what was going well, what was not going well, and where we thought we needed to go next. We audio-recorded the weekly meetings and transcribed them to add to our data set.

We used both ongoing and retrospective analyses of the data, which consisted of our daily memos and meeting transcriptions. The ongoing analysis, which occurred during the two semesters we were teaching, observing each other, and writing memos after each lesson, was the basis for continued reflection on our teaching and learning about our teaching, the testing of emerging hypotheses, and the strategies for promoting further development of the PTs' mathematical understandings. During the retrospective analysis, we examined the larger corpus of data through a carefully structured review of all the relevant data.

We began coding our data at the end of the first course. We used open coding (Corbin & Strauss, 2008) on the memos and meeting transcripts from the first 2 weeks of the semester to identify themes that emerged from that data. After looking at the data individually, we met and compiled a list of the themes that we had identified. Using this list, we then individually looked at 5 weeks' worth of data to see if the codes that we had identified matched our data. We met together again and as a group refined our list. During this meeting, we compiled a list of code definitions

and examples, so that we were using the codes consistently. Since we were using open coding, we looked for any themes that emerged from the data, but as we developed our codes and definitions, we were able to categorise many of them around aspects of MKTT.

8.5 What Did We Learn?

Through coding our data, we discovered evidence that working in our CoP had helped us to develop our MKTT (the goal of our joint enterprise) in a number of ways, through our meetings and observations (our mutual engagement) and our reflections, memos, tasks, and revised lesson plans (our shared repertoire). As teacher educators, we developed (a) an enhanced understanding of the mathematics content of elementary school mathematics (MTE CCK and SCK), (b) a deeper understanding of the ways in which young adults (who may or may not already be familiar with the material) learn and how that affects our teaching and planning (MTE KCS), (c) different ways of questioning and facilitating students' problem-solving (MTE KCT), and (d) a better understanding of ourselves as MTEs through reflecting on both our own actions and those of others. Our mutual engagement and shared repertoire helped us achieve the goal of our joint enterprise.

8.5.1 Mathematics Content Knowledge

As former mathematics majors, we entered into teaching these courses feeling confident with the elementary school mathematics material. However, we soon realised that much of this material was more complex than we had originally thought. Ma (1999) described elementary mathematics as fundamental. She wrote:

> Fundamental mathematics is elementary, foundational, and primary. It is elementary because it is at the beginning of mathematics learning. It is primary because it contains the rudiments of more advanced mathematical concepts. It is foundational because it provides a foundation for students' further mathematics learning. (p. 124)

As teacher educators, we had not looked at some of the mathematics since we were in elementary school ourselves, and at that time, we were not studying it with the depth, breadth, and thoroughness that we would need in order to teach it and to help PTs develop the deep understandings that they would also need in their teaching. The following example on counting methods illustrates one instance where our mutual engagement in our CoP helped us develop mathematics content knowledge.

Thinking Differently About Counting Methods One activity in our curriculum involved using combinations and permutations to determine the number of possible outcomes in a given situation. Having taught this activity before, we discussed the observation that our students did not seem to develop a deep understanding of

permutations and combinations. Instead, they learned the words and where the keys for permutations and combinations were on their calculators, and then they punched the appropriate keys and got an answer. Alternatively, if we did not allow them to use their calculators, they substituted numbers into a formula, but did not understand where the formula came from and why it was appropriate to use in a given situation. In discussing our plan for this lesson in our weekly meeting, we decided that rather than defining permutations and combinations during class, we would try to get our students to think about the situations in the problems and use the fundamental counting principle on every problem. If the order did not matter in a certain situation, they would need to divide their answer by the total number of orders. In the memo that follows, Dana describes how she introduced the problem: *At State University, a group of seven students wishes to select a committee of four to negotiate student activity fees with the Dean of Students. How many committees can be selected from the group of seven?*

> In the past, we have given notes for this activity, but ... this time, I didn't even really define permutation and combination in terms of their formulas. I talked about the first question [7C4] as we have 4 slots, so we have $7 \times 6 \times 5 \times 4$ ways of picking people to fill them, but then we have to divide by the number of ways of arranging 4 people. I'm not sure how it will work, but I liked it better than them just punching in 7C4 on their calculators. (DO, Memo 10/31/07)

At first, this method of talking about combinations seemed to be much better than what we had done in the past. The students really seemed to be thinking about whether or not the order mattered in a problem and when they needed to divide by the number of orders to figure out the answer. However, when Dana was asked a question about a homework problem, *An experiment consists of tossing a coin 8 times and observing the sequence of heads and tails. How many different outcomes have exactly 3 heads?*, she found it difficult to explain the problem without using the words "choose" or "combination". Here is an excerpt from her memo following this discussion:

> In my head, I knew the answer was just 8 choose 3. You have 8 coins and you want to choose 3 of them to be heads. This makes total sense to me and I usually teach it that way. However, since I am not doing "choose", I had a lot of trouble explaining it...I still have trouble working out why you would draw three slots instead of 8. It sort of makes sense, but I can't really explain it. I just don't know what the best way is. (DO, Memo 11/05/07).

Through talking with members of the CoP and reflecting through her memo (part of our shared repertoire), Dana was able to eventually make sense of how to explain the problem both to herself and to her students. Had she not had the benefit of the CoP, it would have been easy for her to go back to encouraging the PTs to use formulas. Patrick summarised this idea in a memo:

> The temptation here is very high for both the instructor and the students to fall into using the formulae for factorial, combination and permutation in which case the students may not get to critically think about what they are doing. The instructor needs to consciously work on avoiding getting trapped into that so that he/she can lead the students into actually thinking about why they are doing what they are doing. (PM, Memo, 10/31/07)

Joanna later mentioned the same thing in a CoP meeting where we were discussing how the CoP had been beneficial. "I think I hadn't thought about before, about how to teach, some of these ideas without referring to combinations and permutations" (JM, Meeting Notes, 1/31/08).

As this example illustrates, it can be difficult to teach in a way that supports students in developing deep, connected understandings. In general, we found that elementary school mathematics had a number of complexities of which we were unaware before we began teaching mathematics content courses for PTs and reflecting deeply on our teaching through our CoP. Like the majority of US teacher educators teaching these classes, we did not have experience teaching elementary school ourselves (Masingila et al., 2012). Through our mutual engagement and shared repertoire (e.g. reflections, memos, tasks, lesson plans), we came to realise the importance of looking deeply at the underlying mathematics behind the representations, algorithms, and definitions that we use, and we believe that understanding elementary school mathematics with strong levels of depth, breadth, and thoroughness is an essential component of MKTT. Being able to discuss and work through the challenging content with the CoP allowed us to expand our own MKTT (our joint enterprise) by learning new mathematics ourselves and discussing ways to encourage PTs to work with the content with high levels of depth, breadth, and thoroughness.

8.5.2 Working with Young Adult Learners

As teachers of young adults, we had to be cognisant of how their learning may be different from how children learn content. We also had to be prepared with the idea that these PTs had probably seen much of the material before, and we needed to think about how to build on their prior knowledge and prompt them to engage with the ideas meaningfully. For example, the majority of our students enter their mathematics content classes knowing how to add, subtract, multiply, and divide whole numbers. However, for most of them, this means being able to perform an algorithm, mostly without thinking, rather than knowing why the steps of the algorithm work. In order to make arithmetic operations more meaningful for our students, we require them to add, subtract, and multiply using bases other than ten, so that they cannot merely rely on the procedures that they have ingrained but instead must think about how the processes of addition, subtraction, and multiplication work. The following example shows the challenges that we faced when introducing multiplication in different bases.

Introducing Multiplication in Different bases Kazima, Pillay, and Adler (2008) describe designing meaningful "first encounters" with mathematical ideas as important tasks for teachers. They define these first encounters as "the first moment of the didactic process or process of study" (p. 285) and stress that these first encounters should be purposefully designed. In terms of this task for teacher educators, most of the encounters that prospective teachers have with the mathematical topics in their

content courses are not "first encounters". Thus, the job of the teacher educator is perhaps to provide a "deeper encounter" with the mathematics, or a first encounter into looking at the underlying mathematical features of a topic.

After successfully using base blocks to add and subtract with regrouping, our students were ready to move on to multiplication. The problems the students were asked to solve were $23_{four} \times 30_{four}$, $45_{six} \times 32_{six}$, and $78_{nine} \times 234_{nine}$. Joanna decided that she wanted her students to understand the role of place value in multiplication, so she would introduce multiplication using the partial products method: e.g. $45 \times 32 = (40 \times 30) + (40 \times 2) + (5 \times 30) + (5 \times 2)$. However, while this method had its advantages, it had some drawbacks as well, as evidenced by this excerpt from her memo:

> I hesitated with trying to decide whether to get into a deeper discussion of what larger than single-digit multiplication is when discussing multiplying two-digit numbers in bases other than ten. In the end, I did go into the meaning of multiplication using partial products. I think this did prompt the students to think more deeply about multiplication and their talk showed this, but it also slowed them down and was more difficult. Tradeoffs! (JM, Memo 9/12/07)

After watching Joanna teach her class (part of our mutual engagement), Dana also considered using partial products to introduce multiplication. However, in the end, she decided instead to use the standard algorithm, with which her students were already familiar in base ten, and translate it to the different bases:

$$\begin{array}{rrrr} & & 4 & 5_{six} \\ & \times & 3 & 2_{six} \\ \hline & 1 & 3 & 4_{six} \\ + \; 2 & 2 & 3 & 0_{six} \\ \hline 2 & 4 & 0 & 4_{six} \end{array}$$

She discussed her thought process in choosing an algorithm with which her students would be more familiar, since the different bases would already provide a challenge for her students, and the base block manipulatives had limitations when multiplying a two-digit number by another two-digit number.

> I think that the advantages to using partial products is that conceptually it makes a bit more sense. You can think about having 3 flats, 4 times and it makes sense. However, as we found, it doesn't make sense to think about 3 flats times 4 flats—at least pictorially. Another disadvantage to using partial products is that it is further away from the algorithm that they are used to. (DO, Memo, 9/12/07)

In the meeting following these lessons, we had a long discussion (part of our mutual engagement through our CoP) of the advantages to introducing multiplication in different bases using each algorithm. Below are some excerpts from the discussion where Dana pointed out that Joanna had used partial products while she chose not to:

Joanna … the real problem that I see is that they don't know which column it goes in.

Dana A couple of reasons that I thought it made it harder is, one, it gives them a lot of things to add. … I liked it because it makes more sense conceptually. It makes multiplication more meaningful, at first, like when you're talking about, okay, four longs times two, that makes sense. But then when we're, when we …

Joanna Longs times longs.

Dana We got into the issue with longs times longs, and that doesn't really make sense, and ...

Patrick You're forming rectangles or squares ...

Dana I think that, it's a deeper level of mathematics, and that, to think about multiplication in that way, is a good idea. …

Patrick For people doing it for the very first time, it's hard.

Dana To give them the algorithm that I think they're more familiar with, my students, for the most part seemed to struggle less with it.

Joanna Okay.

Dana It was like, once they got it, they were like, yes, I remember how to multiply, and the mistakes that they were making, were, just transferring to the base a little bit.

Joanna Because in fact, this, the point of this is to have them do it in another base ... In Chapter Three, we'll talk about what the algorithm means.

Dana So, I'm going to wait until Chapter Three ... but, the way that [Joanna's students] were talking about it, I thought was more, deeper mathematically, or more mathematically mature than the way that mine were talking about it. … mine, learned the algorithm, and so now they can do it, and they understand how to do it, whereas, yours were saying, okay that's one unit and two longs.

Joanna So, there's a tradeoff (Discussion Meeting, 9/13/07).

Having the CoP allowed us an opportunity to think about and discuss the pedagogical decisions that we made. Being able to watch and read reflections about multiple groups of students and approaches gave us a chance to observe and discuss how the students reacted to the content. We decided that both methods had their advantages and disadvantages and that while we were both happy with the method that we each chose, in the future, we would think about how to incorporate both methods, in order to gain the advantages of each.

8.5.3 Thinking About Our Questioning

We taught our mathematics content courses through problem-solving (Lesh & Zawojewski, 2007; Schroeder & Lester, 1989). This meant that rather than presenting mathematics directly to students, we presented them with problems that engaged

them with mathematical ideas through participating in the problem-solving process. Therefore, it was important that, as MTEs, we thought about how we facilitated the problem-solving and the questions we were asking our students to help them in the problem-solving process.

As novice MTEs, it was helpful for Dana and Patrick to observe Joanna – an "old timer" (Lave & Wenger, 1991) – as she facilitated her own students' problem-solving activities and also to have her observe them while they taught. Using our memos and discussions (our shared repertoire) provided an opportunity for us to think about how we were interacting with our students and develop our KCT. The following excerpts from our memos illustrate some of our reflections on questioning.

> Jo, who was observing today, said that there were times when I asked questions and then did not wait very long before giving answers when there was silence. (DO, Memo, 2/4/08)

> One thing I noticed is that Jo tends to give an assignment and then hang by the front or leave the room for a few minutes before going to help her students. I generally start walking around as soon as I give the assignment. I think that Jo's method is probably good to get them talking to each other about the problems without asking questions right away. (DO, Memo, 10/22/07)

> It really makes me think about how much help we give the students versus how much they actually need. If that had been my class, I probably would have said something sooner, and Sarah wouldn't have been able to come to the same realisation—or at least not on her own. (DO, Memo, 11/26/07)

> Sometimes I think by helping them it encourages helplessness. I think that next year I will work more at hanging back a bit more like Jo does. (DO, Memo, 5/5/08)

> [My student's] question got me wondering if I have made a pattern of questioning them only on their incorrect responses. This is something that I need to pay more attention to and just be more aware about my questioning. While there is a general tendency to ask them questions about their incorrect solutions, I think it is also helpful to question them on their correct responses to ensure that they are making the connections intended by the activities. (PK, Memo, 2/13/08)

Through their observations of Joanna and our CoP discussions, both Dana and Patrick were able to reflect on their questioning. They noticed that Joanna often seemed to give her students more independence to struggle through problems and make mistakes, whereas they were more apt to try to help right away. Joanna's role as an old-timer (Lave & Wenger, 1991) in the CoP and the shared repertoire that we had established allowed Dana and Patrick to view their teaching practice in comparison to hers. Additionally, they were able to think about when they were probing our students' thinking and work on asking them to share their ideas, both when they were correct and when they were incorrect. Over the course of the academic year, Dana and Patrick both strove to better use their questions to encourage their students to become independent thinkers and doers of mathematics.

8.5.4 Learning from Our Community of Practice

Through our CoP, we developed a true community where each member felt comfortable asking questions of each other and of ourselves. Through our shared repertoire, we were able to experience multiple perspectives on teaching mathematics content to PTs, and we discussed these perspectives with others who were mutually engaged in the pursuit of developing MKTT (our joint enterprise). We have all found that participating in this study has changed us and helped improve our practice as MTEs. As a group, we think more about clearly communicating our goals and decisions to ourselves, to our fellow MTEs, and to our students. We all think about the mathematics content at a much deeper level, what makes certain topics challenging, and how do certain ideas/activities provide building blocks for others. We focus more on the process of helping PTs become mathematical problem-solvers, rather than worrying about covering all the material of elementary school mathematics (which is an impossible task.) Below, we highlight how each of us has personally benefited and learned from the CoP.

Dana Participating in the CoP helped me to develop my pedagogical content knowledge, specifically knowledge of content and students. Seeing how students in both Joanna's class and my class interacted with the material gave me twice as much data to look at. But the most important part was the reflecting part – writing down what happened with my students and thinking about how to help them construct knowledge and see problems with their work helped me make connections and figure out ways to help my students in the future. Being able to share the experience with the other members of the CoP also helped me develop my own knowledge in a way that reflecting on my own would not.

Additionally, I am much more comfortable with the material these days than I was when I was teaching during our CoP. I think this comes from experience and also from realising the importance of knowing what I am doing. It was clear from my reflections that on the days when I was not well prepared, the lesson did not go well. I think the original assumption was that elementary school mathematics is easy and that I should not really need to work hard to teach the content, but clearly we figured out that this was not true, and it has encouraged me to make sure that I am prepared for every lesson with a deep understanding of the material.

I think that a lot of the things I do with my students now come as a result of participating in the CoP. At this point, I have been teaching mathematics content courses for prospective teachers for more than 10 years, and many of the things I focus deeply on now were things that I struggled with and learned about from participating in the CoP. An example of this involves having PTs use models to represent operations:

> Other places where people had issues were that some people wrote that taking 1/3 of a number was subtraction, and a few still are having trouble modeling multiplication. They model the answer to the problem rather than the action taking place in the problem. I talked about this by drawing a rectangle divided into 12ths on the board. I shaded in one square

> and asked what was being modeled. A student replied that it was 1/3 times 1/4. I explained that just shading in 1/12 of a rectangle was a model of 1/12 or 1/12 of 1, but not of 1/3 times 1/4, or 1/4 times 1/3, or 1/2 times 1/6. If they wanted to model the process of 1/3 times 1/4 then they needed to do something to indicate the 1/3 and the 1/4, not just model the answer. A number of students in my class are struggling with this, and I have noticed this from students in Jo's class as well. (DO, Memo, 4/14/08)

In reading over this memo, I realise that this focus on modelling is something that I still do today. For example, I tell my students when we model something like 12 – 7 = 5, that we need to be able to see 12, 7, a model of subtraction, and 5 in the model. This idea is something that I developed from my reflections and work in the CoP.

Additionally, I still use CoPs in my teaching and research today. With the model of my original CoP as a guide, I have formed a new CoP with MTEs at institutions across the United States. Our joint enterprise is the creation, implementation, and modification of cognitively challenging tasks for mathematics content courses for PTs (e.g. Thanheiser et al., 2013). We meet regularly to develop a shared repertoire of work on our tasks and discussions of their implementation, as well as to support each other's teaching and research.

Patrick One of the main things I learned from this CoP was the importance of having PTs create their own understandings. I learned to support my students' development by carefully leveraging students' initial ideas to build a profound understanding of the concepts taught in these courses.

Something else I learned from the CoP was the importance of thinking carefully about the objective of the lesson. The CoP was designed to facilitate reflective practice about the efficacy of our efforts to engage PTs in thinking carefully about mathematics. That meant that we interrogated our teaching practices to determine how well they were aligned with this goal. For example, in this memo, I was documenting my reflection on the challenge of supporting PTs to think carefully about counting techniques:

> I think counting concepts are probably too complex for these students to be able to get a good understanding by doing one activity. While I am not advocating teaching counting by giving the students formulae and a significant number of practice problems, I think the students could benefit from a better selection of counting problems and a better sequencing of the problems. Also, as previously discussed in one of our meetings, the temptation here is very high for both the instructor and the students to fall into using the formulae for factorial, combination and permutation in which case the students may not get to critically think about what they are doing. The instructor needs to consciously work on avoiding getting trapped into that so that he/she can lead the students into actually thinking about why they are doing what they are doing. (PK, Memo, 10/31/07)

Having the opportunity to discuss these challenges through our mutual engagement in a CoP enhanced my development as a MTE.

As a result of my participation in the CoP, I am more reflective about my practice of preparing prospective teachers. While I no longer write memos like I did during the CoP, I find myself reflecting about how my students are experiencing my class

and what I could do to afford them opportunities to deepen their understanding. This all started with my experiences in the CoP:

> I believe the co-teaching/observation experience and writing a memo is really helping me reflect on how I can make this course a better course for the students. By reflecting on the students' struggle, the goal of the activity and my actions as an instructor combined with my observation in Jo's class, I am getting an opportunity to think about my teaching more than I would normally have done. (PK, Memo, 10/29/07)

In the years since the completion of this research work, I have continued to engage in professional CoPs. For example, as the Developmental Education (Dev. Ed.) Lead for my department, I organised the Dev. Ed. instructors to form a CoP to discuss issues around Dev. Ed. The CoP met once a month and engaged in reading and discussing research literature about best practices in Dev. Ed. While the structure and goals of the Dev. Ed. CoP were different from the MKTT CoP, I was able to leverage my MKTT CoP experiences to facilitate the Dev. Ed. CoP.

Joanna I benefited from the mutual engagement of having other people to think carefully about how to support PTs in understanding the mathematical concepts underpinning the procedures they would be teaching in the future. For example, the CoP with Dana and Patrick caused me to rethink how I engaged PTs in thinking about tasks involving probability:

> I wonder why we do probability first before methods of counting. I find these activities very frustrating because of their disjointedness and lack of providing students with tools, such as tree diagrams, for solving problems. What I think we need to do is, after the semester is over, think carefully about what probability and statistics topics should be covered in the chapter and how they should be ordered. (JM, Memo, 10/31/07)

I also learned by observing Patrick and Dana teach and saw some things that they did (e.g. how Patrick engaged his students in thinking about necessary and sufficient conditions for definitions of plane figures) that provided me with new insight into my own teaching practices. The CoP discussions afforded the opportunity to rethink how the tasks were structured (e.g. Dana suggested that we change the order of the conditions given in the task where students attempt to construct triangles given the lengths of three sides, part of our shared repertoire) and to rethink teaching practices: "I've been challenged to think more deeply about my teaching practice, why I do what I do, when to change what I do, etc." (JM, Memo, 5/6/08).

In general, I found the opportunity to work closely with other colleagues teaching the same courses to be very helpful in my own thinking about the tasks and the courses: "Our preparatory meetings help me to clarify in my mind the big picture that I have for the day's lesson, and our weekly debriefing meetings help me as I form the bigger picture in my mind about the course" (JM, Memo, 2/4/08).

I have changed my practice as a result of participating in the CoP as I am more intentional in approaching my teaching through a stance of inquiry. I have implemented the changes in curriculum that we discussed during the time of the CoP (e.g. teaching probability through counting rather than as permutations and combinations) – implementing the shared repertoire we developed. Additionally, each year I

work with instructors and interns in the courses to create a CoP where we meet in person twice weekly during the semester and use technological tools such as email, Blackboard™, and OneNote™ to share our thinking and planning related to the tasks and the PTs' learning and engagement.

Overall, the three of us all feel that we have benefited greatly from participating in the CoP, and we all currently participate in CoPs and use many of the ideas and the knowledge that we gained in our teaching. Although our study involved intense work, we believe that it was worth it to improve our teaching, to develop our MKTT, and to become more reflective practitioners overall.

8.6 Communities of Practice in the MTE Community

While the three of us believe that participating in the CoP was a worthwhile experience, we also understand that not everyone in the mathematics teacher educator community has access to a group at their own institutions. We suggest some questions and answers for people interested in forming their own CoPs based on both our own work and other successful examples of CoPs that focus on MTEs in the research literature.

Question How can MTEs work together to develop their mathematical knowledge for teaching teachers and improve their teaching?

Answer We believe that the most important part of developing a CoP is a group of willing and committed participants who have a desire to improve their teaching through mutual engagement around a joint enterprise. We suggest a group size of two to six people working towards a common goal of improving their teaching in some way. While it is possible for larger groups to work, we believe that a smaller group size is more beneficial to forming an actual community. Active participation in a CoP requires dedication from each of the members to play a role in the community and work to make change. Only if each member is dedicated to the group will a CoP be able to develop trust to question each other and offer constructive feedback. Along these lines, members of a CoP must feel safe participating. Although Joanna was Dana and Patrick's advisor and mentor, we were able to create an environment where we could talk to each other as equal members of the community who were all working to learn together.

Additionally, we believe that a successful CoP would require a way for the members to meet regularly (mutual engagement). The meetings would not need to be in person but would need to involve all of the members. The members of the CoP should be working towards a common goal (joint enterprise), so we suggest that all members of the community be teaching common course content and have common

learning goals (Morris & Hiebert, 2013) at the time of the CoP. For our CoP, we were all teaching exactly the same course (other examples of this include Hiebert, 2013; Hiebert, Morris, & Glass, 2003; Van Zoest et al., 2006), but we envision that this could work for others who are teaching similar content over the course of a year or semester (e.g. Goos & Bennison, 2008).

Finally, in order to engage in a successful CoP, we believe that there must be a plan for documenting what you are doing (developing a shared repertoire); otherwise there will not be opportunities for meaningful change because the members will not remember what happened. We suggest that writing reflective memos is an important part of a CoP, in order to have a record of what happened, in order to put one's thoughts and ideas on paper (or screen) to help solidify thinking, and in order to share one's ideas with other members of the group. Alternatively or additionally, we audio-recorded and transcribed our CoP meetings, which helped achieve the purposes above. Similar types of data collection are found in Hiebert et al. (2003) as well as Van Zoest et al. (2006).

Question What are different models of a network of support for MTEs, including MTEs who are the only person at their institution teaching particular courses?

Answer We suggest that groups interested in participating in CoPs collaborate with others in small groups either inside or outside their institutions. Twenty-first-century advancements in technology have made it possible for self-formed teams to communicate within and across organisations distributed geographically. In particular, high-speed internet access, availability of sophisticated video conferencing capabilities and free or cheap file-sharing capabilities have greatly enhanced the ability of people to network and to collaborate professionally. This redefined context for learning, where participants can contribute, share, and learn in a virtual community of practice (VCoP), or "third space" (Hulme, Cracknell, & Owens, 2009, p. 539), has facilitated and extended community dialogue and reflection beyond the confines of physical meetings. Proponents of the "third space" theory argue that these spaces facilitate dialogue among VCoP participants that is safe, secure, and supportive (Hulme et al., 2009. A third space such as a virtual community of practice can be used "to create a community of practice and shared reflection on common experience around what the professionals do together" (Hulme et al., 2009, p. 541).

Additionally, Goos and Bennison (2008) present an example of an online CoP that began as part of a course but continued on after the students had graduated and began teaching in schools full time. The initial face-to-face interactions of the CoP helped its members develop trust and mutual engagement to allow them to continue their joint enterprise virtually. The mathematics education community through conferences, meetings, and workshops has the ability to establish groups such as this one that could continue virtually.

8.7 Conclusions

Overall, we found that participating in our CoP was extremely worthwhile for our development as MTEs. While the work that we did took a lot of time and effort, we believe that the benefits and opportunities for growth, both individually and as a group, outweigh the challenges. While it is possible for MTEs to reflect on their teaching alone by writing memos and examining student work, they would miss out on the benefits of the CoP. Without forming a CoP, there is no opportunity to reflect on the actions of others, to receive feedback on your teaching, or to see other ways of doing things. Additionally, participation in a group provides accountability for its members.

We would like to call on the Mathematics Education Community to encourage its members to form CoPs by providing a Special Interest Group or forum for interested people to meet with others who share this interest. Improving the teaching of mathematics for future teachers will benefit the community and the population at large.

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